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(REPRINT, MICHIGAN ENGINEER, 1904, p. p. 229 to 245.)

PROPAGATION OF HOT AND COLD WAVES DOWN INTO THE EARTH.

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(*a*) *Surface Temperatures.*

The variations of the temperature at the surface of the soil may be taken as the result of the combination of a number of different variations of temperatures. These latter will differ in amplitude, that is to say, in range of temperature due to them. They will also differ in period, that is, the time which elapses between the occurrence of the greatest divergence, and the recurrence of the same, that is, between successive maxima. Two of the most prominent periods are the day and the year. They will also differ in the date at which the first maximum takes place, after any date that we take from which to begin to reckon our time, say midnight of January 1. It may be shown by Fourier's theorem that no matter how complicated the variation of temperature, it may be considered as the sum of a series of such harmonic variations,[1] each being expressed mathematically as —

$$(1)\quad T = c \cos\left(2\pi \frac{t}{p} - l\right)$$

In this expression —

T is the divergence of temperature from the average, eliminating this particular variation, at a time t.

c is the amplitude; that is, the maximum divergence from the average.

p is the period for the particular term, the length of time between maxima.

π is 3.1416, but in estimating angles $2\pi = 360°$, so that $2\pi \frac{t}{p}$ is an angle which is to four right angles as the time for which T is sought is to the time between maxima.

l is the value $2\pi \frac{t}{p}$ assumes at the time of the first maximum, and the time of it after the date of the beginning is $l. \frac{p}{2\pi}$

[1] Byerly, Fourier's Series, Chapter II.

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Now, at the depth, which we will call x, the temperature, so far as it is due to the surface temperature, may be represented as the sum or set of vibrations having the same periods, but the maxima will be less and later.[1] Each will be expressed mathematically as —

$$(2)\quad T_x = c\ e^{-R} \cos\left(2\pi \frac{t}{p} - l - R\right)$$

In this expression —

T is the temperature of a point at the depth x at the time t as before.

c, π p, t, and l have the same meanings and values as in expression (1).

e is the exponential base 2.71828.

R depends on the retardation of the maxima.

The retardation of the maxima depends on the depth, the period, and the capacity of the ground to diffuse heat. This latter we will call K.

K is by Byerly called a^2 and by Kelvin $\frac{k}{c}$. It is proportional to the rate of change of temperature with the time at a given depth. It is also inversely proportional to the acceleration of the change of temperature for different depths at the same time; i. e., $D_t u = K D_{x^2} u$. K is usually assumed to be invariable, though this is not strictly true. and in such case if the temperature was proportional to the depth, it could not change with the time.

$$(3)\quad R = x\sqrt{\frac{D_{x_2} u}{D_t u}\ \frac{\pi}{p}} = x\ \sqrt{\frac{\pi}{K.p.}}$$

As to the units, x and $\sqrt{dx^2}$ are referred to the same unit, also dt and p, d^2u, and consequently a change in units does not effect R, nor does a change in temperature unit from degrees Fahrenheit to degrees Centigrade affect K. K is the conductivity and c the thermal capacity per unit volume, which is for water, 1; for most rock, about .5; and for air, .000307.

Obviously the diffusivity, or spread of heat, when the strata have to be heated up as the heat wave spreads, will be relatively less than the conductivity or flow of heat when the flow has become steady, if the body absorbs a good deal of heat in being heated up. One must be careful not to confuse the conductivity and the diffusivity, as if often done; and in fact I myself, in my Annual for 1901, page 247, have mixed some conductivity

[1] See Byerly's Fourier's Series, Art. 69, much simplified here.

data with diffusivity, as writers are sometimes obscure. The conductivity may be defined, for instance, as the number of calories, or heat units, which an indefinite slab, without varying the temperature, will transmit per square centimeter through a slab a centimeter thick, the difference of temperature of the two sides being 1°, while the diffusivity is the conductivity divided by the thermal capacity per unit volume. This is the specific heat per unit weight multiplied by the specific gravity. The former is calculable from the chemical composition, and is for most rock-forming minerals not far from .20. For water, it is 1.00. The specific gravity for rock-forming minerals is usually between 2.5 and 3.5, consequently the specific heat capacity per unit volume of the strata, except for porosity, is near .5 to .6. If the pores are filled with air, the heat capacity per unit volume is diminished; if filled with water, it is increased. Yet though c is increased, wetting adds so much to the conductivity of strata that it adds to their diffusivity also, so far as experiment has gone.

It may be shown that the greater the time between maximum heat or cold, the greater the period of retardation of the maxima will be at a given depth. It will be as the square root of the period.

For maximum values of T, $\cos\left(2\pi\frac{t}{p}-1-R\right)=1$, the greatest value a cosine can have, then $1+R=2\pi\frac{t}{p}+2n\pi$, the *first* maximum (when $n=o$) being at the date $\frac{1p}{2\pi}$ at the surface, and at the date $\frac{(1+R)p}{2\pi}$ for the depth x. The difference $\frac{Rp}{2\pi}$ is the time it takes the heat to penetrate the distance x, which is from equation (3)

$$\frac{x}{2}\sqrt{\frac{p}{K\pi}}=\frac{Rp}{2\pi}$$

That is, variations of long period move in more slowly, but they go more deeply than those of short period. For when we come to consider the amplitude (which is the original amplitude multiplied by $e-R$, i. e. $(0.36788)^R$ which is always a fraction decreasing from 1 as R increases from 0), we find that the amplitude decreases with the depth also, but more

slowly the better the diffusivity and the longer the period of variation.

The diffusivity (K) in materials such as those of which the earth's crust is composed, usually varies from .001 to .01,[1] if the unit of time is a second, and of distance a centimeter.

In the progress of fluctuations of temperature from the surface they soon tend to smooth out minor irregularities as they go down, and if not strictly harmonic, become so. Now for any one particular heat wave, say that due to day and night, as they go down —

(1) The times of maxima are retarded by an amount which is to the period (p) of the heat wave (in the instance cited 1 day) as R is to 2π where $2\pi = \frac{66}{7}$ and $R = x\sqrt{\frac{\pi}{Kp}}$

(2) The amplitude of vibration is to the original amplitude as $1 : e^{-R}$; i. e., is $(.36788)^{-R}$ of the original. We can for any one fluctuation take $p = 1$ if we choose a proper value of K.

TABLE OF K FOR DIFFERENT UNITS OF DEPTH AND PERIOD.

	Second.	Hour.	Day.	Week (7 days).	Month (1-12 year).	Year.
Centimeter	1 (a)	3600	86400	610800	262800	31536000
Meter	.000001	.0036	.0864	.6108	.2628	31536
Inch	.154	556	13700	93600	40600	4870000
Foot	.001073	3.87	92.8	684	2830 (b)	33800 (c)

(a) Units of B. O. Peirce, Callendar, and McLeod.
(b) Units of Angstrom.
(c) Units of Kelvin.

Suppose, for instance, the unit of time is a day, and of distance a foot, and let us ask to what distance from surface does the daily variation cause a fluctuation of 1 per cent of its own amount, if the diffusivity is .01, in centimeters and units, i. e., .928 in foot-day units —

Amplitude $= c\,(0.36738)^{-R} = .01c$
Canceling and taking logarithms.
$R\,(-.4343) = -2$. Therefore $R = 4.6$.

As the variation is daily, if we let $p = 1$, $R = x\sqrt{\frac{\pi}{K}}$

[1] Annual Report 1901, page 247. B. O. Peirce. Rocks like traps have a diffusivity of about .006 to .007. The Calumet conglomerate's is as high as .01, which is taken as a rough average of underground strata by Lord Kelvin.

Taking the diffusivity K for these units from the table as .928, we find that $R = 1.84\ x$.

Then combining the two values of R, x is about 2.3 feet. The soil diffusivity will be usually from 2/3 to 1/5 of this, and since the daily variation of the surface soil temperature is not usually over 30° to 40°, the daily variation is usually less than a degree Fahrenheit, even at less than 2.3 foot below the surface.

It is plain, however, that when we come to consider thawing frozen ground, or vice versa, an enormous amount of heat will be absorbed in thawing the same out. The diffusivity may be easily ten times less. On the other hand, a warm rain may carry the temperature down very rapidly.

For instance, I made the following temperature observations on the peat bog of Mud Lake, Sec. 1, T. 1 S., R. 5 E., in Washtenaw County, July 16, 1903: —

	Fahrenheit.
Temperature of air in shade	86°
Temperature of air in sun, surface of bog	117°
Temperature at 1/2 inch depth	94°
Temperature at 2 inches	73°
Temperature at 4 inches	63° or 62°
Temperature at 12 inches	57° to 56.4°
Temperature at 18 inches	57°

At other places in that same bog the same day, we found at different times: In sedge marsh, at 3 inches depth, 64°; at 4 inches depth, 50 feet within shade of Tamarack belt, 64° F.; in Cassandra belt, 3 inches below water level, 56°, and at 8 inches, 56½°; 2 feet deep in peat, 56.3°; away out on the floating bog, at 4 inches depth, 68°; at 18 inches, 66°. Now the maximum temperatures that day for adjacent stations of the weather service were 80° to 82°, and the minimum the previous night were from 46° to 55°.

July, 1903	14	15	16	17	Ave. July.	Ave. Aug.
Ann Arbor	72 —52	74—45	79 —51	81—57	82.5—60.2	76.0—56.2
Ypsilanti	71 —50	72—44	76 —55	81—58	80.5—58.2	75.0—54.9
Average	71½—51	73—44½	77½—53	81—57½	81.5—59.2	75.5—55.5
Average for day	61	59½	65	69	70.4	65½

The range of soil temperature for the day could hardly have been over $(117-65)\ 52 \times 2 = 104°$. The amplitude at 1 foot

and 1.5 foot would then be at a diffusivity of .01 c.g.s.; i. e., assume K = .928 for foot and day; i. e., p = 1 —

$R = x\sqrt{\frac{\pi}{.928}}$ units being day and foot.

= for 1 foot depth, 1.84.
= for 1½ foot depth, 2.76.
Amplitudes corresponding will be: —

log. 104 =	2.017	
for 1 foot depth — 1.84 (.4343) =	— 0.80	
for 1½ feet depth — 2.76		— 1.20
logarithm of amplitudes,	1.217	0.817

15.9° F. and 6.35° F. respectively and the maxima or the minimum would be reached 29/100 of a day and 44/100 of a day later, respectively, than on the surface, and a greater range means a greater diffusivity. To get a temperature of almost 10° below the daily mean at a foot, as we have, we should have to assume a lower minimum soil temperature than is probable, and greater diffusivity than is really likely. Finally, the possibility of the temperatures at a foot and a foot and a half diverging from the daily mean 10° on account of the cold the night before, utterly breaks down, in view of the relative constancy of the temperatures both in depth and at various times of day.

We see then that the temperatures are practically unaffected at a foot's depth by the daily variation, the amplitude of which at that depth is reduced to not over 2°, and the diffusivity (c.g.s.) will be about .004 to .005. From considering the temperature at 1 foot as fixed by the annual variation in temperature we find that the greatest summer temperature at a foot depth would be nearly thirty-seven days later than at the surface. This leads to an extremely low diffusivity, like that of pumice or dry sand, and shows what a good nonconductor peat may be taken to be. The result may, however, be affected by evaporation and consequent depression of temperature, the peat acting like the wet flannel wrapper of a Hindoo water bottle.

(100° to 24°) $e^{-R} = 2°$—, and hence —

R (—.4343) = log. 2—/104 or log. 2/24, according to the amplitude of the surface variation taken = — 1.1716 + or — 1.079 + ∴ R = 2.96 or 2.48 + ∴ if x = 1 and p = 1 since $R = x\sqrt{\frac{\pi}{Kp}}$ $K = \frac{\pi}{R^2+} =$ 0.355, or 0.5 in day-foot units = .0038 or .0056.

Supposing that the temperature at 1 to 1½ foot depth 56.6° is fixed by the annual variation (it is probably 2° or 3° too low on account of previous cold days), which is, or should be, just about a maximum (70°.6 daily mean) at the surface on July 17, reckoning the time from that same date last summer (1 = 0), taking this time p = unity = 1 year (the diffusivity will accordingly be referred to years and feet), we have,—

$$T_x = (56°.6 - 46°.6) = 10° + = (c = 24°) = (70.6 - 46.6)\, e^{-R}$$

$$(\cos\ 2\pi \frac{t}{p} - 1 - R)$$

$$= 24 \cos\ (2 - R)\, e^{-R} = 24 \cos R/eR$$

$= \log. 2.4 + \log. \cos R - R$ ($\log. = .4343$) $= -.38$. This equation is nearly satisfied if $R = 37°$ ($\frac{37}{360}$) $2\pi = .646$ in foot-year units. $\frac{37}{360} \times \frac{365}{} = 37$ days, which is the difference between maxima at the depth at which 56°.6 is taken and at the surface. The maximum mean daily temperature at a foot depth is over a month later than at the surface. And if $R = .646 = x\sqrt{\frac{\pi}{K\,p}}$ $\therefore K = \frac{\pi}{R^2} = 7.6$ in year-foot units = .0002 in c.g.s.

Suppose we have observations at intervals of one fourth of the variation in temperature apart, i. e., in the case of the daily variation 6 hour intervals, we have, calling this —

$T_{1x} + M, T_{2x} + M, T_{3x} + M, T_{4x} + M$, M being the average temperature.

$$T_{1x} = ce^{-R} \cos\ (2\pi \frac{t}{p} - 1 - R)$$

$$T_{2x} = ce^{-R} \left[\cos\ \left(2\pi \left(\frac{t}{p} + 1/4\ p\right) - 1 - R\right) = \sin\ \left(2\pi \frac{t}{p} - 1\ R\right)\right]$$

$$T_{3x} = c\ e^{-R} \cos\ 2\pi \left(\frac{t}{p} + 2/4\ p\right) - 1 - R = -\cos\ \left(2\ \frac{t}{p} - 1 - R\right)$$

$$T_{4x} = c\ e^{-R} \cos\ \left(2\pi \left(\frac{t}{p} + 3/4\ p\right) - 1 - R\right) = s\ in\ 2\pi \frac{t}{p} - 1 - R$$

The sum of two observations a half period apart will give us the mean average temperature. Then we can get T_{1x} T_{2x} etc., readily. Squaring the sums, we get —

$T^2_{1x} + T^2_{2x} = c^2 e^{-2R} (\cos^2 + \sin^2 = 1) =$ the square of the amplitude at the depth x, from which we can readily find R, if we know c. Or if we have a series of different observations at different depths, we can eliminate between them. Also dividing —

$\frac{T_{2x}}{T_{1x}} = \tan\ (2\pi - 1 - R)$, from which we can also compute R.

There are thus two ways of computing the diffusivity of heat. In the former case we estimate from the decrease in amplitude or amount of maximum variation. But if we have a series of timed observations, we can also estimate the diffusivity from the time at which this maximum temperature occurs for various depths.

Between 1888 and 1892, measurements of the soil temperatures at different depths were made at Grayling and at Lansing.[1]

These were read three times a day, at 7 A. M., 1 P. M., and 7 P. M.; i. e., six-hour intervals, but not in the winter. Therefore they do not help us as regards the blanketing effect of the snow in winter, and a study of the annual variation must be very imperfect. We can, however, get some comparative results as to daily variation and as to diffusivity, and apply the formula above; and it is well to notice also that the composition and porosity of the Grayling soil has been tested, as well as its diffusivity.[2]

Throughout we assume that we can isolate pure harmonic vibrations corresponding to the single curve. This is not strictly true, but the farther from the surface the more the curves tend to become such.

The addition of morning and evening observations should give us twice the daily mean, free from the daily variation.

$$\frac{T_{7\text{ A. M.}}}{T_{1\text{ P. M.}}} = \frac{T_{7\text{ P. M.}}}{T_{1\text{ P. M.}}} = \tan \overline{1+R.}$$

From this we derive $1+r$, which is proportional to the time after 1 P. M. that the daily maximum arrives. Finding this for various depths, and remembering that $R = x\sqrt{\frac{\pi}{k}}$ and that we know the depths, and the period (p) is taken as 1, we shall have expressions with only l and K, so that if we have these observations for two different depths we can find both l and K. Taking, for instance, the observation for September, 1891, at the Agricultural College, we have: —

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[1] Report State Board of Agriculture, 1888, pages 154-164; 1889, pages 85-101; 1890, pages 143-152; and 1891, pages 92-97.

[2] Bulletin State Board of Agriculture, Bull. 99, page 11.

Depth.	7 A. M.	1 P. M.	7 P. M.	7 A. M.—7 P. M.	Mean.
3 in.	61.3	72.3	69.2	130.5	65.25
6 in.	63.6	67.2	68.1	131.7	65.85
9 in.	64.8	63.8	66.5	131.3	65.65
12 in.	65.1	65.1	65.7	130.8	65.4
24 in.	64.3	64.4	64.3	128.6	64.3

Average.....65.29

If there were only daily harmonic variations to account for the column of means should be identical. The variation in the means is due to sine terms of a period longer or shorter than a day. With the exception of the 24-inch thermometer, which is distinctly lagging in temperature, owing to the annual variation, the variation is less than $\frac{1}{4}°$.

We have, accordingly, as variations from the means for the various depths: —

Depth.	7.	1.	7.	$\frac{T\ 7\ P.\ M.}{T\ 1\ P.\ M.}$	$\frac{l+R \text{ in degrees}}{15° = 1 \text{ hr.}}$	$1+R$	$\frac{L+R}{x}$	$\left(\frac{l+R}{x}\right)$	K in inch. & day units	K in units	cE	$-R$
3	—3.95	7.05	3.95	.561	29° 20′	.51	.17	.0288	105	.0083		8.1
6	—2.25	1.35	2.25	1.65	58° 40′	1.02	.17	.0288	105	.0083		2.62
9	— .85	—1.85	.85	—.458	156° 20′	2.75	.30	.09	35	.00275		2.03
12	— .3	— .3	.3	—1.	135°	2.55	.20	.04	78.5	.0062		1.34
					10 P M.							
24	— .0	+1.1	+.0	+0.	360° or +180°	+3.14	261×2 +13	.068	46	.0036		1.1

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From the two shallowest depths we infer that the hottest part of the day at the surface was almost exactly at 1 P. M. and $l = 0$. The diffusivity will vary between .003 and .008. Apparently it takes about four hours for the daily heat wave to penetrate half a foot, but nearly twenty-four hours for it to penetrate two feet.

Let us now apply this same analysis to July, 1888, at Grayling: —

Depth.	7 A. M.	1 P. M.	7 P. M.	7 A. M.—7 P. M.	Mean.
3 in.	63.92	75.49	75.52	139.44	69.72
6 in.	66.52	71.10	74.06	140.58	70.29
9 in.	68.28	68.97	71.79	140.07	70.02
12 in.	68.26	68.02	69.74	138.00	69.00
24 in.	66.80	66.77	66.60	133.40	66.70

The variation in the means shows the importance in influence of periods with others terms than 1 day. In the first three depths the difference is not over ¼°. Obtaining Tx we have: —

x	(7 A. M.)	(1 P. M.)	(7 P. M.)	$T_{\frac{7}{1}} \div T_{\frac{7}{1}}$	$(1+R)\frac{360}{2\pi}$	$1+R$	$1+R\frac{24}{2\pi}$ [1]
3 in.	—5.80	+5.78	+5.80	+1.003	45°.30	.79	3h.2
6 in.	—3.77	+0.81	+3.77	+4.65	77°.50	1.35	5h.4
9 in.	—1.74	—1.05	+1.77	—1.69	90 + 31°.10	2.12	8h.1
12 in.	—0.74	—0.98	+0.74	—0.75	180°–37°.45	2.49	9h.55
24 in.	—0.04	+0.07	—0.10	—1.43	360 –55°	5.31	20h.04

We obtain, using the values at 3 inches and 6 inches to determine l, $l = 2(.79) - 1.35 = .23$, indicating that the warmest part of the day at the surface was nearly an hour after 1 P. M. The values for R will be: 57, 112, 189, 226, and 508 respectively; and of $R/x = \sqrt{\frac{\pi}{k.p.}}$ .19, .19, .21, .19, and .21 respectively, a fairly uniform rate averaging .020 in inch-day units, which will lead to a diffusivity of 78.5 in inch-day units, or .006 in c.g.s.

For amplitudes we find the greatest divergence, $\sqrt[2]{T^2_{x7} + T^2_{1x}}$ successively, 8.°17, 3.°36, 2.°30, 1.°23, and 0.71° respectively. If we assume the maximum divergence from the average at the surface is 15°, we shall obtain for values of R: .61, 1.33, 1.88, 2.50, and 3.06, and for R/x .20, 0.22, 0.21, and 0.14, which agree quite fairly (except the last, where the amplitude at two feet is so slight as to be greatly affected by the error of observation, and may be affected by the yearly variations.

Thus we may conclude that the mean diffusivity at the point where the observations were made at Grayling was on that July close to .006, the average surface temperature between 69° and 70°, while the average daily range of the soil temperature was 14°.6 above and below; i. e., from 56° F. to 84° F.

We have also tried one other month, April, 1889. The results were as follows: The mean range of temperature at 3 inches was greater than that of the air, so the surface range of soil temperature may have been much greater. The whole variation was not nearly so great as in July. By taking the

[1] Time that maximum is retarded.

differences from the mean value of the temperature at any two times six hours apart, and squaring them, we obtained the square of the greatest difference at that depth as before, and accordingly found at 3, 6, 9, 12, and 24 inches respectively, maxima of 8.1, 2.62, 2.03, 1.34, and 1.21 respectively. From these again the diffusivities may be computed. If we assume R = 51 for 3 inches, we find a maximum at the surface of 13.3 above the mean, the mean daily variation being about 27°, which harmonizes well with atmospheric data, and we find for successive values of the retarding factor, R, values of 1.62, 1.9, 2.3, 2.5, respectively. Then R/x will be .17, .27, .21, .20, .11, respectively, or an average of .18, or for the diffusivity in units per day, .98, or in c.g.s. units, .007. The average from the retardations of the maximum was .006.

We see, then, that records of the temperature at different depths give us check methods of determining the diffusivity. Even three observations a day, at intervals of six hours, will help us out pretty well. And with the diffusivity determined, we can readily figure the amplitude of a heat or cold wave at a given depth, and how long it will take to get down there.

This subject is not a very important one, I imagine, and my only reason for bringing it before you is that I find very little about it either in your proceedings or others; but there are times, however, when the extent of frost into the ground is a matter of at least curious interest, and there may be some time when the thing might come up in the way of theory. Mr. Riggs, for example, was speaking to me of a case where he noticed the frost was down over a path about thirty inches, and in the field about fourteen inches, and that is why I bring it to your attention.

Mr. Taft: How do you read those thermometers at nine inches?

Mr. Lane: Dr. Callendar used an electric thermometer. The wires ran up to his ammeter; anyway his electric thermometer was arranged in his laboratory.

Mr. Brigden: This thing may strike in a practical way. In some work I have had to do, we have laid our surface pipe five feet for several years, and then we changed to five and a

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half because once we found that they froze in cold weather at five feet. Some cities put their pipes as low as six feet. I wonder if any of those observations were taken in such a place as a street, where the frost penetrates to such a distance as you have mentioned; and whether the frost penetrates to a greater depth almost into the spring, when it is thawing above, and should be, according to our ideas, thawing from below; and why it is the frost goes down freezing almost completely. The cold penetrates from the ground into the pipes, or can it get into the surface?

Mr. Lane: There you have that annual hot wave or cold wave, which works down into the ground, and your pipes will freeze, although it is commencing to thaw up above. The first man that brought this to my attention was Professor Wright, who traveled in Siberia, and made some experiments out there, and I worked the matter out for him in an approximate way. Now the main thing, if possible, is to arrange to have a substance with low diffusivity — the main thing is not to pack your stuff too much, over your pipes, but have a dry clay top. I have noticed in the Upper Peninsula that they are burying their water pipes one and one-half feet or about that, but they count on a blanket of snow; if the wind scoops it off, they get into trouble.

Mr. Brigden: We sometimes have to go into fields where there is a blanket of snow to thaw our pipes, but that is very rarely the case.

Mr. Lane: It seems as if it would be worth while, where you had special trouble at a particular point, to have a couple of thermometers or so, and see if you could not trace up the matter. While the theory is comparatively easy, and I don't think the observation would be difficult, still we have not quite enough facts. By trying daily observations, as I say, we will find that the diffusivity will vary from day to day, according to the amount of moisture in the soil.

Mr. Brigden: Is there anything known about the thawing of deeply frozen earth, five or six feet deep, as to whether it occurs in the spring; whether it comes up or goes from the surface; whether the frost comes up or goes down?

Mr. Lane: You can not work that out by this theory entirely. I will tell you what happens. There is a band of frost

starts in the winter; it works down, and gets narrower and narrower as it goes down, and the excess above freezing point above continually diminishes, and finally it so deepens that at five or six feet it will peter out, and be only a little above freezing.

Mr. Brigden: If you uncover iron substances, would it be any different?

Mr. Lane: The greater the diffusivity, the further it will go.

Mr. Brigden: How does a blanket of leaves affect it?

Mr. Lane: The more porous and more dry the material, the less the diffusivity.

Mr. Brigden: There is not much thawing from underneath; it is almost entirely from the surface.

Mr. Lane: It works mainly down.

Mr. Taft: I have heard some say that before the spring thaw, the thaw from the top by the sun shining and warming up above, made the frost strike deeper; and if there came up a warm rain, it would draw the frost out more.

Mr. Lane: Just as much as the rain itself works down, runs down, and carries the heat with it. That is all. A warm spring rain does make a very great difference; but it is because the water diffuses the heat down directly.

Mr. Taft: We were thawing a sewer recently, and we had about eighteen inches of frost in the street. The day before Christmas we had a rather warm rain, and I was surprised to find that almost all of the frost was drawn from the ground; and some of the wise weather prophets, local weather prophets, said it was owing to the rain, and that if we had had snow, it would have made the frost strike deeper.

Mr. Lane: The difference is this; in that case the snow merely thaws superficially, and the cold wave moves on down. The rain changes the whole thing; and where the diffusivity is about .00156, a good fine rain brings it up to .37.

Mr. Hodgman: Did I understand Mr. Lane to answer to the effect that the frost does not thaw from beneath upwards?

Mr. Lane: Only very slightly.

Mr. Hodgman: I know there is a good deal of belief to the contrary. When there comes a lot of snow on to the wheat, people go there and find that the frost has gone, and they believe that the frost has come out from underneath.

Mr. Lane: There you have a different thing. You have the surface of the ground changed, for one thing.

Mr. Holmes: I did not quite get that experiment of the burying of the water pipes in the time of that thaw; in other words, we get frozen pipes, pipes frozen in the basement at the time of a thaw, the bursting of these pipes at the time of the increase of the heat. It is a common occurrence that at the time of a thaw the pipes will burst.

Mr. Lane: I will answer Mr. Holmes first, and will then return to the other. Even after the thawing begins, the frost may be working down a little bit. The heat wave may still be working down a little, but soon after the thaw the new heat wave will overtake it, the pipes will then melt, and you will find the water comes out from pipes that have been cracked before. As soon as they are frozen solid,— it does not make any difference how much burst they are,— you will not have any flow of water; you have got to have them first frozen and then thawed, as most of us know to our sorrow in our own domestic arrangements. It is very often in the morning, when the house heats up, that the trouble will occur.

Now that blanketing effect of snow is very important, and it is one worth considering from an agricultural point of view. You have, for example, a depth of snow come on something like that [blackboard drawing], the shape of the heat wave or the cold wave. We will call this the positive direction of heat, and this the direction of cold. Now that wave will go on down and die out; you can only keep your ground frozen by a continual supply. Of course in that case that would be the shape of the curve; the cold is going in there all the time, and it will be a long period of cold wave, and the ground is frozen down just this way. Now suppose that you have a blanket of snow put on there, that wave proceeds right on down, and this temperature below will drop off; but if there was more cold going in, it is going in from this surface, and if it is a snow surface, it is a nonconductor. I wish we had some good figures on the diffusivity of snow, but we have not got them. Now this wave in the meantime will go on working down, and the top of the ground will thaw out; but in the course of time the whole of it will thaw out. I think that answers the real point of the case. For example, if your ground was frozen.

and you had a fall of snow, if you should dig down there directly after your snow came, you might find a little layer thawed out and then a frozen layer, and then the unfrozen ground still farther down.

Mr. Walker: Does it draw from the other side?

Mr. Lane: No, it keeps on working down.

Mr. Hodgman: Where does the heat come from? Where there is a blanket of snow, you say the cold wave passes along down, and of course there is a warmer wave following it that causes it to thaw out. Where does that come from when we have the blanket of snow?

Mr. Whitney: I think if there is a little thawing, the snow begins to melt, and it has the effect upon the top of the earth of a warm rain, and the frost is drawn out.

Mr. Lane: Very frequently a snow comes when the temperature is above freezing; not always, but there is quite a proverb in the Upper Peninsula, where I have had some chances to observe, that the snow that lasts all winter comes on a wet ground; and there it follows the amount of heat that can be observed is very large, and the capacity of water for heat is very large. What becomes of the cold wave is this: it has to spread out, and heat up the strata; and as it spreads out, we get the dying-out effect, as I have spoken of; and, moreover, I have assumed in this diagram — which I should not assume — that the zero point of that heat wave was at freezing point; it is not so, it is at the mean temperature of the place.

Mr. Brigden: I can hardly give up the idea of having some of the thawing done from below. You take a manhole with an iron cover; and one who has run a water works knows that the frost is in the ground to a depth of four or five feet; and you take the cover from this manhole, and you will find a rim of frost down several feet.

Mr. Holmes: I am interested in this same question that Mr. Brigden brought up, and have made some tests with the ordinary self-registering thermometers. We made this case of tile four feet deep, put the eighteen-inch false cover below, and sealed the top, put the self-registering thermometer in the top of that case and in the bottom of that case, and we never got the temperature below 40° in the lower part under this second cover eighteen inches below the surface of the ground, and that

was the hardest winter that we have ever had in Grand Rapids. We had our water pipes frozen there in a good many places at five feet. These cases were only four feet, and they evidently did not freeze below the bottom of the tile; but for the space above this second cover, or just above the eighteen-inch, there would be an inch of frost around inside of this top case that was sealed from the air, and there is no doubt but that the frost that winter went at least three feet, and probably more than that, below the surface of the ground. There was not a meter in there, but there was a bottle of water which, if it had frozen, would have let us know. It, with the self-registering thermometers, gave us the temperature as near as we could get it.

Mr. Lane: You got a greater variation with the upper thermometer of the two?

Mr. Holmes: O, in the upper case we got below a freezing point.

Mr. Lane: I do not know just how to account for these facts. You have here around a meter box a good many pipes, and you have a number of things that I do not know anything about. It is a fine chance for investigation. I still believe the frost works down until it dies out.

Mr. Taft: I think that probably the reason for our seeing frost in the manhole is that the bricks are dense; they are a good conductor of cold from the outside, and the moisture in the manhole rising, condenses on these bricks, and the intense cold of the brick freezes it right there. Probably the temperature of the brick farther away, right at the same level, would be warmer, and that is the reason we see the frost there.

Mr. Walker: About the time that it commenced freezing here, sometime over two months ago, we commenced laying some water pipes in a town. The frost had just commenced setting in the ground so that it formed a little scale. There was some on the ground at that time, and the ground has not been without a cover of snow. The weather kept getting colder and colder, and still we kept on with our work digging and laying pipe, and the frost kept going down. We found it in this shape to-day, while the snow is still on the ground; all the time the frost kept going down, and we kept digging, until when we quit work, quit laying pipe about two weeks ago, there were

places where there were eighteen inches of solid frost, which we had to cut right through or melt. There were other places in the same street where there were not six inches of frost. I was there last week, and they are putting in the telegraph poles, and they did not average three inches of frost.

MR. TAFT: Where they put the telegraph poles is right by the side of the street, where there is a considerable amount of snow.

NOTE

The report of discussion from page 13 is rather inadequate, as there was a general conversation with blackboard illustrations and two or three questions at a time,— hard to take. But I have attempted to correct the report as printed in the Engineer, and it shows at any rate some of the practical points for investigation.— L.